A Matching Approach for Line Planning

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1 Introduction

Public transportation systems are usually designed within a sequential process: After deciding about stations and tracks, the lines are planned, followed by a timetable, vehicle schedules and drivers' schedules. In this paper we deal with line planning in combination with timetabling. We analyze how lines can be improved without changing a given timetable, and use this in an iterative approach for improving both, the line plan and the timetable. Line planning and timetabling have both been extensively studied in the literature, see Schöbel (2012) for a survey on line planning and Liebchen (2008); Kroon et al (2009) for success stories in timetabling.

2 A matching approach for line planning

Let a public transportation network PTN = (V, E) with its set of stops and direct connections E be given. A line in the PTN is a path, and the line concept is the set of all lines \mathcal{L} together with their frequencies $f_l, l \in \mathcal{L}$. A (periodic) timetable assigns a departure and an arrival time to each line at each station. This time is repeated periodically.

Assume that a line concept together with a timetable is given. Our goal is to improve the lines, preferably without changing the timetable. The general idea we follow is a local improvement strategy: We consider one single station s, see Figure 1. Every line l_i which passes through this station is cut into two parts: the arrival part l_i^{arr} and the departure part l_i^{dep} . A line l_j that ends or starts at the station contributes only with one part l_j^{arr} or l_j^{dep} , respectively. We now want to improve the line plan by a new assignment of the arrival parts of the lines to the departure parts of the lines. To this end we set up a matching problem with matching costs c_{ij} for every assignment between an arrival part



Figure 1: A station which is passed by four lines which can be re-assigned.

 l_i^{arr} to a departure part l_j^{dep} . These matching costs are chosen depending on the objective function we wish to follow:

Minimizing the number of transfers: A transfer between two lines is always inconvenient for the passengers. Minimizing the number of transfers is hence an important goal which also contributes to the robustness of a public transportation system. In order to reduce the number of transfers locally, we set the matching costs c_{ij} to the number of passengers who arrive with l_i^{arr} and depart with l_j^{dep} . We then find a matching with maximal weight. This assignment minimizes the number of transfers locally.

Maximizing the number of direct travelers: The number of direct travelers has been extensively studied, e.g., in Bussieck et al (1996). Note that minimizing the number of transfers is not the same as maximizing the number of direct travelers, also if we consider a reassignment of lines at one station only. If we want to improve the number of direct travelers we define the matching costs c_{ij} as the number of passengers who do not have any transfer on their journey if they need not change from line *i* to line *j* in the considered station. The assignment given by a matching of maximum weight locally maximizes the number of direct passengers.

Minimizing the traveling time: The traveling time is an objective function of line planning which is on the one hand considered as important for the passengers, but on the other hand hard to compute, see Schöbel and Scholl (2006). One might ask if the matching approach is also able to improve the traveling times. This is not the case. If the timetable and the passengers' paths are fixed, the traveling times for the passengers are independent of the particular assignment chosen. and hence cannot be improved by a reassignment of the lines.

Minimizing the costs of the line plan: From the perspective of the public transportation company the costs are an important factor when planning a public transportation system. The costs of a line concept are influenced by many parameters such as the length of the lines, the time needed to run a line, or the number of cars of a train, see Claessens et al (1998). In order to improve the costs of a line concept, there are different possibilities: First, one can minimize the number of cars needed. To this end, we define c_{ij} as the maximum of the number of cars of line l_i and the number of cars of line l_j , i.e., in the re-assignment we try to assign line parts to each other with a similar number of cars. Another possibility is to look at the overall costs of the line concept and define $c_{ij} = f(l_i^{arr}, l_j^{dep})$ as the costs when assigning l_i^{arr} to l_j^{dep} .

Maximizing the robustness: For a robust solution it is preferable that passengers have enough time for transferring between lines. In order to guarantee this we consider for every arrival part l_i^{arr} of a line the transfer time $t_{ij} = (t_j - t_i) \mod T$ to all possible departure parts. If we assign l_i^{arr} to l_j^{dep} the robustness costs c_{ij} are set to the minimum of all other transfer times, i.e.,

$$c_{ij} = \min_{k \neq j} t_{ik}.$$

We then minimize the largest robustness costs in the matching or the sum of all costs in the matching. Recall that we know that the traveling time of the passengers is not changed by reassigning the lines (see above). Hence, this approach improves the robustness of a line plan without changing the traveling times of the passengers, i.e., without adding additional slack times.

3 Applications

The matching procedures described so far are applied at one single station. In order to improve line plans, we look at several stations sequentially, and finally also iterate between line planning and timetabling. These settings are described next.

First, a given line plan can be improved according to each of the above mentioned criteria. This can be done in an iterative way by cutting the lines in one station, finding a better reassignment and then proceeding with another station. The procedure stops when no local improvement is possible any more. Note that this algorithm is finite whenever we have a global improvement of the solution (which is the case for maximizing the number of direct travelers, or for minimizing the number of transfers) or whenever the reassignment in one station is independent of the assignment in other stations (which is the case for maximizing the robustness). Convergence, however, cannot be guaranteed for all possible cost structures.

Another way to use the improvement algorithm is to include it in an iterative approach for the integration of line planning and timetabling. Here, the goal is to improve both, the line plan *and* the timetable. Our matching procedure is one possible way to improve lines when a fixed timetable is given. This can be used in two steps in which we alternate between improving the line plan and the timetable iteratively until no improvement is found any more. This may be done following one or several objective functions hence leading to a locally optimal (Pareto) solution.

The results for the improvement are considered on close-to real world data using the LinTim (2015) library. We will show convergence of the matching approaches and the improvements within a procedure for integrated line planning and timetabling.

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