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Periodic or aperiodic timetables? The case of a single line
Extended Abstract

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Abstract From a passengers' perspective there are two types of timetables: periodic and non-periodic ones. Periodic timetables are repeated, e.g., every hour and provide a reliable and easily memorable service. Aperiodic timetables schedule the vehicle trips irregularly over the day. In this work we compare periodic and non-periodic timetables analytically. Since periodicity is an additional constraint, an aperiodic timetable can be better adapted to the demand. We quantify these effects on the simple example of one single line in which it is possible to compute the best periodic and aperiodic timetables. Our results allow conclusions also for more general transport networks.

Keywords timetable design · periodicity · single line

1 Periodic and aperiodic timetables

The question if a timetable should be periodic or non-periodic is an ongoing discussion. Periodic timetables which are repeated, e.g., every 60 minutes, can be memorized easily and provide a reliable service. In contrast, non-periodic timetables can be adapted better to the demand by taking into account opening and closing times of shops, office hours, or schools. The question we discuss here is: How much time can be saved for the passengers by having aperiodic timetables instead of periodic ones?

There is plenty of literature on timetabling. For periodic timetabling see Odijk (1998); Liebchen (2006) for classic literature, and Borndörfer et al

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(2017); Schiewe and Schöbel (2020) for integrating the route choices of passengers. For aperiodic timetabling, also called timetable synchronization, we refer to, e.g., Robenek et al (2018); Liu and Ceder (2018).

Unfortunately, both periodic and aperiodic timetabling are hard to solve, see, e.g. Schmidt and Schöbel (2015). We hence discuss their differences in a simplified setting in which we can solve them exactly.

2 Timetabling along a single line

We consider one line and assume that the vehicles (buses or trains) are big enough for all passengers, and that the costs correspond to the kilometers operated. To obtain roughly the same costs for the periodic and the aperiodic timetable we fix the number of vehicle trips in both instances.

As demand we consider OD-pairs with preferred departure times, i.e., a list consisting of triples

$$(s_i^p, s_j^p, w^p) \text{ for all passengers } p$$

where s_i^p is the start station, s_j^p the destination station and w^p the preferred departure time, the *wish-time*, of passengers p. Assume a passenger wants to start her journey at time w^p, boards a vehicle at station s_i at time $\pi_i > w^p$ and reaches her final destination at time π_j at s_j. Then her journey time is $\pi_j - w^p$ where $\pi_j - \pi_i$ is the traveling time and $\pi_i - w^p$ is the waiting time at the station. In the objective function we sum the journey times over all passengers.

Furthermore, let the times $L_{i,i+1}$ for driving between stations s_i and s_{i+1} and the dwell times $L_{i,i}$ at s_i be fixed. The traveling time t_{ij} from station s_i to s_j is then constant,

$$t_{ij} := L_{i,i+1} + \sum_{k=i+1}^{j-1} (L_{k,k+1} + L_k).$$

Hence,

- we only have to fix the departure time of all vehicle trips at the first station.
- The sum of all traveling times of the passengers is constant, i.e., only the waiting times are relevant.
- The destination stations s_j^p can be neglected, only the start stations and the wish-times are relevant.

We prove that both problems are *location problems* which are polynomially solvable:

Theorem 1 *The problem of finding an* aperiodic *timetable can be reduced to the (directed) p-median problem along a line.*

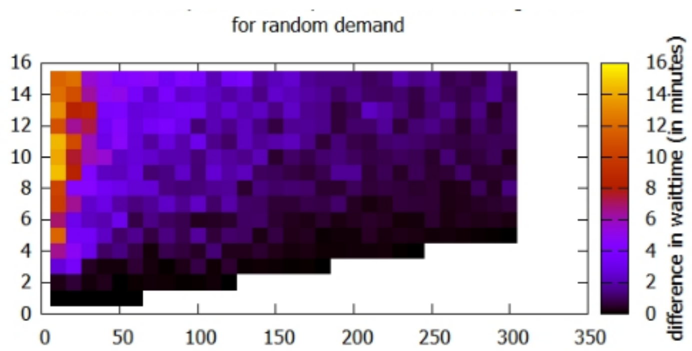

Fig. 1 Heatmap showing the difference of waiting times (minutes per passenger) between periodic and aperiodic timetables for up to 300 passengers (x-axis) and 1–15 vehicle trips. Deep purple/black colors indicate that the periodic timetable is almost as good as the aperiodic timetable.

The directed p-median problem can be solved polynomially, see Jackson et al (2007).

Theorem 2 *The problem of finding a* periodic *timetable can be reduced to the (directed) p-median problem along a circle. It is polynomial.*

3 Comparison

The above analysis allows to run a large set of experiments in which we determine optimal periodic and aperiodic timetables and compare their waiting times for different demand structures. Figure 1 shows an example for random demand. The highest differences are obtained for few passengers. The reason is that an aperiodic timetable can adapt well to few wish-times (if we have less passengers than vehicles, every passenger has a vehicle exactly at her wish-time!), while a periodic timetable must offer one trip every 60 minutes and hence achieves higher waiting times.

4 Conclusion

The outcome depends on the structure of the demand: the more the demand is periodic, i.e., it behaves similar in each period, the less are the differences between periodic and aperiodic timetables. Random demand becomes periodic for many OD-pairs. The recommendation hence is to use periodic timetables in case of unknown, high, or periodic demand.

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